

ISBN : 978-602-96426-0-5

Total Pages : 1170



PROCEEDING of

IndoMS International Conference on Mathematics and Its Applications (IICMA 2009)

Yogyakarta - Indonesia, October 12th - 13th 2009

**Published by
IndoMS (Indonesian Mathematical Society)**

Secretariat :
Department of Mathematics
Faculty of Mathematics & Natural Sciences
Gadjah Mada University
Kampus Sekip Utara
Yogyakarta - Indonesia, 55281
Phone : +62 - 274 - 552243 ; 7104933
Fax : +62 - 274 555131
Copyright © 2010

Proceeding Team

Editors

Atok Zulijanto, Fajar Adi Kusumo, Budi Surodjo, Ch. Rini Indrati
Indah Emilia Wijayanti, Irwan Endrayanto

Technical Support

Dewi Kartika Sari, Karyati, Susiana, Putri Mahanani

Layout & Cover

Parjilan

Referees

Algebra

Prof. Zhao Dongsheng NIE- NTU Singapore
Prof. Sri Wahyuni Universitas Gadjah Mada
Prof. Pudji Astuti Institut Teknologi Bandung
Ari Suparwanto Universitas Gadjah Mada

Analysis

Prof. Hendra Gunawan Institut Teknologi Bandung
Yudi Soeharyadi Institut Teknologi Bandung
Supama Universitas Gadjah Mada
Ch. Rini Indrati Universitas Gadjah Mada
Atok Zulijanto Universitas Gadjah Mada
Prof. Mashadi Universitas Riau
Prof. Soeparna Darmawijaya Universitas Gadjah Mada

Applied Mathematics

Prof. Widodo Universitas Gadjah Mada
Prof. Edy Soewono Institut Teknologi Bandung
Lina Aryati Universitas Gadjah Mada
Rieske Hadiyanti Institut Teknologi Bandung
Prof. Franz Kappel Graz University
Edy Cahyono Universitas Haluoleo
Hartono Universitas Negeri Yogyakarta
Fajar Adi Kusumo Universitas Gadjah Mada

Computer, Combinatorics and Graph

Prof. Edy Tri Baskoro Institut Teknologi Bandung
Tri Atmojo Universitas Sebelas Maret
Kiki Aryanti Sugeng Universitas Indonesia
Saib Suwilo Universitas Sumatera Utara
Khabib Mustofa Universitas Gadjah Mada
Edy Winarko Universitas Gadjah Mada

Mathematics Education

Abdur Rahman As'ari Universitas Negeri Malang
Utari Sumarmo Universitas Pendidikan Indonesia
Yansen Marpaung Universitas Sanata Dharma
Jaelani Universitas Negeri Yogyakarta
Prof. Suryanto Universitas Negeri Yogyakarta

Statistics

Prof. Subanar Universitas Gadjah Mada
Prof. Budi Nurani Universitas Padjadjaran
Danardono Universitas Gadjah Mada
Gunardi Universitas Gadjah Mada
Muhammad Syamsuddin Institut Teknologi Bandung
I Wayan Mangku Institut Pertanian Bogor

Preface

from President of IndoMS for Proceeding of IICMA 2009:

First of all, I would like to pray for God for His mercy so that we could finish the Proceeding of IICMA 2009 (IndoMS International Conference on Mathematics and its Applications 2009) hold on October 12th - 13th, 2009 at the Departement of Mathematics Gadjah Mada University Yogyakarta Indonesia. On behalf of the IndoMS (Indonesian Mathematical Society), I would like to say congratulation to all authors in the proceeding.

IndoMS or formerly known as “Himpunan Matematika Indonesia” is a forum for mathematicians and users of mathematics as well as people who have interest in enhancing mathematics in Indonesia. The Society is a scientific, nonprofit, non-governmental and professional organization. It was established on July 15th, 1976 in Bandung, West Java. The objectives of the Society are to enhance and extend mathematical knowledge, extend education in the Mathematical sciences, and to increase the role of mathematics in Indonesia. In 2009 IndoMS has 1.151 members consisting of university teachers, mathematicians, statisticians and mathematics-education researchers from 30 Indonesian universities, and school teachers from elementary and high schools. IndoMS has established 8 provincial officers to stimulate and enhance mathematical activities in the country. The branches are branch Special Territory of Yogyakarta and Central Java, branch Banten, Special Territory of Jakarta, and West Java, branch East Java, branch South and West Sulawesi, branch South Kalimantan, branch South Sumatera, branch Nanggroe Aceh Darussalam and South Sumatra, and branch East Nusa Tenggara. Since 1976, IndoMS has already 14 times organized National Conference in Mathematics and National Congress. The next National Conference in Mathematics and National Congress will be held in Manado State University, North Sulawesi on June 30 – July 3, 2010. Since 2006, IndoMS also has already 3 times organized National Conference in Mathematics Education. The next National Conference in Mathematics Education will be held at the Yogyakarta State University in 2011.

Starting in 2009 IndoMS organize International Conferences. IICMA2009 is IndoMS International Conference on Mathematics and its Applications 2009. It is majority supported by Directorate General of Higher Educations (DGHE), Department of National Education, Indonesia through “Professional Organization Symposium Competition Program” (Program Hibah Symposium Organisasi Profesi). IndoMS is one of professional organizations which granted by this program. In this conference we facilitate researchers and users of mathematics to exchange ideas and discuss research results and development of mathematics internationally in the fields of mathematics including mathematics education and its applications.

All 166 full papers in the conference has been reviewed by 36 competence experts. As results of the conference, we got 20 papers are feasible to be published in international journals, 17 papers are feasible to be published in aspirated international journals, 32 papers are feasible to be published in national journals, 65 papers published in this proceeding, and 32 papers are rejected

Finally, I would like to express my sincere appreciation to:

- Dean of Faculty of Mathematics and natural Sciences and Rector of Gadjah Mada University for the permission and cooperation in holding the Conference
- Steering Committee and Organizing Committee for all efforts for the success of the conference

- All invited speakers and all participants from Indonesia and abroad for the active participation of the conference.
- All editors for all efforts to finish this proceeding.

Last but not least, my sincere appreciation is also extended to the DGHE for the major support of the conference.

Yogyakarta, January, 2010
President of IndoMS 2008-2010



Prof. Dr.rer.nat. Widodo

Preface from the Committee

The proceeding of IICMA Conference is a collection of all selected papers that were presented in IndoMS International Conference on Mathematics and its Applications (IICMA) 2009 held at Department of Mathematics – Gadjah Mada University, Yogyakarta, October 12th – 13th, 2009. The selected papers are based on the reviewed results by 36 competence reviewers. Each paper has been reviewed by at least two reviewers. On behalf of the Committee, we would like to say thank you very much to all of the reviewers.

There are 118 papers in this proceeding coming from diverse aspects of mathematics ranging from Analysis, Applied Mathematics, Algebra, Theoretical Computer Science, Mathematics Education, and other related topics. We are sure the papers will inspire, not only writers, but also many other researches in developing mathematics and its applications. Please find the benefit of the proceeding.

Yogyakarta, January 2010
On behalf of the Committee IICMA 2009



Dr. Ch. Rini Indrati, M.Si.
Chair

TABLE OF CONTENTS

	Page
Cover	ii
Proceeding Team	iii
Referees	iv
Preface from President of IndoMS	v
Preface from the Committee	vii
Contents	viii
ALGEBRA	
A Natural Property Of A Boundary Operator On A Simplicial Chain Complex <i>Ema Carnia, Sri Wahyuni, Irawati, and Setiadji</i>	0001-0008
Hereditary Path Algebra <i>Faisal Anwar</i>	0009-0014
Dual Near-Rings and Dual N -Groups (Revisited) <i>Indah Emilia Wijayanti</i>	0015-0022
Quivers of Path Algebra and Path Coalgebras <i>Intan Muchtadi-Alamsyah and Hanni Garminia</i>	0023-0028
Quotient Semigroups Induced by Fuzzy Congruence Relations <i>Karyati, Sri Wahyuni, Budi Surodjo, and Setiadji</i>	0029-0034
Algebras, Coalgebras and State-Based Systems <i>Klaus Denecke and W. Supaporn</i>	0035-0052
A Max-Plus Algebra Approach to Critical Path Analysis in the Project Network with Fuzzy Activity Times <i>M. Andy Rudhito, Sri Wahyuni, Ari Suparwanto, and F. Susilo</i>	0053-0060
Polynomial Over Dedekind Domain <i>Monika Rianti Helmi and Intan Detiena Muchtadi</i>	0061-0068
The Max-Plus algebraic Approach of Reachable Space and Observable Congruence of Linear Discrete Event System <i>Nilamsari Kusumastuti and Ari Suparwanto</i>	0069-0076
Structure Theory of Twisted Toeplitz Algebras <i>Rizky Rosjanuardi</i>	0077-0082
On $\tau[M]$ Cohereditary Modules <i>Suprpto, Sri Wahyuni, Indah Emilia Wijayanti, and Irawati</i>	0083-0094
Hereditary path Algebra and Its Characteristic Trough Injective Module <i>Umi Mardiyati</i>	0095-0100
ANALYSIS	
On Minimum and Maximum of Functions of Small Baire Classes <i>Atok Zulijanto</i>	0101-0106
An Irreducible Continuous Linear Representation of Topological Group and Invariant Space <i>Diah junia Eksi Palupi, Ch. Rini Indrati, and Soeparna Darmawijaya</i>	0107-0112
Quasicontinuous Selection for One to Finite Valued Generalized Continuous Multifunctions <i>D.K. Ganguly and Piyali Mallick</i>	0113-0118

Two Important Extension Theorems for The GAP-Integral <i>D.K. Ganguly and Ranu Mukherjee</i>	0119-0128
Bi-Lipschitz Trivial Quasi-Homogeneous Stratifications <i>Dwi Juniati and Guillaume Valette</i>	0129-0138
Determination of Whitney, Kuo-verdier and Lipschitz Stratification for the surfaces $y^a = x^b x^c + x^d$ <i>Dwi Juniati, Laurent Noirel, and David Trotman</i>	0139-0150
Fractional Integral Operators on Lebesgue and Morrey Spaces <i>Hendra Gunawan and Idha Sihwaningrum</i>	0151-0160
Wavelet Neural Network on Multiresolution Analysis with Particle Swarm Optimization <i>Julan Hernadi</i>	0161-0172
A Construction of Tight Wavelet Frames with Dilation Factor $M > 2$ <i>Mahmud Yunus and Armein Z R Langi</i>	0173-0178
Banach Fixed Point Theorem on M-Fuzzy Metric Space <i>Muhammad Ashar Karim and Ch. Rini Indrati</i>	0179-0188
Modification of Hilbert-Schmidt Operator into the Sense of Banach Space <i>Muslim Ansori, Soeparna Darmawijaya, and Supama</i>	0189-0204
Double and Multiple-Normed Space <i>Soeparna Darmawijaya</i>	0205-0214
Stummel Class of Non-Homogeneous Space Type and Generalized Morrey Space <i>Wono Setya Budhi, Idha Sihwaningrum, and Yudi Soeharyadi</i>	0215-0220

APPLIED MATHEMATICS

The Problem of Acceleration Estimation <i>Alexander Agung S. G</i>	0221-0228
A Mathematical Model of Knowledge-Growing System: A Novel Perspective in Artificial Intelligence <i>Arwin Datumaya Wahyudi Sumari, Adang S. Ahmad, Aciek I. Wuryandari, and Jaka Sembiring</i> ...	0229-0240
Stochastic Model for the Population Dynamic of Anoa (<i>Bubalous sp</i>) in a Farying Multipupulation <i>Asrul Sani</i>	0241-0252
Impact of Perfect Vaccination with Super Infection Mechanism to Pathogen Strain Replacment in an Epidemic Model <i>Bayu Prihandono, Lina Aryati, and Fajar Adi Kusumo</i>	0253-0264
A Semidefinite Relaxation Approach to Solve Uncertain Conic Optimization Problem with Binary Variables <i>Diah Chaerani, Sudradjat, and Firdaniza</i>	0265-0276
The Leslie Matrix in Population Model with Age Structured <i>Dwi Lestari</i>	0277-0286
Numerical Solution of Mathematical Modeling of Tumor Growth with Immunotherapy and Chemotherapy <i>Edwin Setiawan Nugraha, Mustafa Mamat, and Agus Kartono</i>	0287-0300
Modeling the Eradication of <i>Aedes Aegypti</i> with Sterile Insect Technique <i>Eminugroho Ratnasari and Lina Aryati</i>	0301-0312
A 2-D Interpolation Method that Minimizes an Energy Integral <i>Endang Rusyaman, Hendra Gunawan, A.K. Supriatna, R.E., and Siregar</i>	0313-0326
The Reduced Rank of Ensemble Kalman Filter to Estimate the Temperature of Non Isothermal Continue Stirred Tank Reactor <i>Erna Apriliani and D. Adzkiya</i>	0327-0334

Normalisation of a Coupled-Three Oscillators with Energy-Preserving Quadratic Nonlinearity Near 1:2: \mathcal{E} -Resonance <i>Fajar Adi Kusumo</i>	0335-0340
Data Selection with Hessian Matrix <i>Hanna A. Parhusip</i>	0341-0352
Modelling of Total Investment and Its Efficiency in The District of Sidomukti <i>Hanna A. Parhusip</i>	0353-0362
Bicriteria interval Linear Programming <i>Herry Suprajitno and Ismail Bin Mohd</i>	0363-0368
A Method for Solving Multi-objective Linear Programming with Fuzzy Probabilistic Coefficient Objective Function <i>Indarsih, Widodo, and Ch. Rini Indrati</i>	0369-0376
Optimal Downlink Power and Rate Allocation in Multi-Cells CDMA <i>Irwan Endrayanto Alucius, A.F. Gabor, and R.J. Boucherie</i>	0377-0390
Study the Dynamics of Human Infection by Avian Influenza: Case Study in the Central Java Province of Indonesia <i>Kartono, Widowati, and R. Heri SU</i>	0391-0396
Multi-Assets Barrier Options as Unique Viscosity Solution to Hamilton-Jacobi-Bellman Equations <i>Komang Dharmawan</i>	0397-0410
Fully Nonlinear Solutions of Supercritical Flow on Terminated Channel <i>L. H. Wiryanto and Adil A. Akbar</i>	0411-0418
Local-Time Dependence Amplitude of Pc3 Magnetic Pulsations Observed at Biak, Indonesia <i>L. Muhammad Musafar K.</i>	0419-0424
A Numerical Tachnique to Obtain a Scheme of 8th Order Implicit Runge-Kutta Method to Solve the First Order of Initial Value Problems <i>La Zakaria</i>	0425-0434
Local Non-Similarity Analysis On MHD Convective Heat and Mass Transfer Flow Past a Wedge with Variable Viscosity and Thermal Radiation Effects <i>Muhaimin Ismoen, Ishak Hashim, and R. Kandasamy</i>	0435-0450
Stochastic Modeling the Spread Of DHF in a Single Closed and Open Population <i>Mukhsar and Asrul Sani</i>	0451-0464
Ergodicity in an Infinite Measure Space <i>Nanang Susyanto</i>	0465-0470
Stability of Delayed S I R Model with Vital Dynamics <i>Rubono Setiawan</i>	0471-0478
The Time Periodic Damping Coefficient In The Dynamic Of Cable Stayed Bridges <i>S. B. Waluya</i>	0479-0488
Feedback Zero-Sum Linear Quadratic Dynamic Game for Descriptor System <i>Salmah</i>	0489-0498
Ant Colony Optimization Algoritms for the Traveling Salesman Problem <i>Sarwadi and Agus Leksono</i>	0499-0508
Parametric Excitation in a Self-Exited Three-Degrees of Freedom Problem <i>Siti Fatimah</i>	0509-0516

Fuzzy State Feedback Control with Multi-Objectives <i>Solikhatusun</i>	0517-0524
Solving of Degenerate Cauchy Problem Via the Alternative Form on Factorization Problem <i>Susilo Hariyanto, Lina Aryati, and Widodo</i>	0525-0534
Stability Analysis and Maximum profit of Predator-Prey Population Model With Time Delay and Constant effort of Harvesting <i>Syamsuddin Toaha</i>	0535-0546
Description and Modelling of Hybrid Power Systems <i>Tiryono Ruby and Volker Rehbock</i>	0547-0560
Mathematical Modeling and Analysis of Ammonia, Nitrite, and Nitrate Concentration: Case Study in the Polder Tawang Semarang, Indonesia <i>Widowati, Hermin PS, and Sutimin</i>	0561-0570
A Mathematical Model for the Spread of Avian Influenza: Spread from Bird to Human <i>Yuni Yulida and Lina Aryati</i>	0571-0578

COMPUTER, GRAPH AND COMBINATORICS

Implementation of Hidden Markov Model in Clustering of Sequence Protein and Its Improvement Using Prior Knowledge <i>Afiahayati, Sri Hartati, Sri Mulyana,</i>	0579-0588
Critical Set of Edge Magic Total Labeling of Cycle Plus 2 edges Graph <i>Chairul Imron and Suhud Wahyudi</i>	0588-0594
Super Edge Antimagic Total Labeling of Disjoint Union of Threeungular Ladder and Lobster Graphs <i>Dafik, Slamain, M. Fuad, and Riris R.R.</i>	0595-0606
Construction of Edge Consecutive Edge Magic Total Labeling on a Dis- connected Graph <i>Denny Riama Silaban and Kiki A. Sugeng</i>	0607-0612
On γ -Labeling of Wheels and Fan Graphs <i>Diari Indriati and Mania Roswitha</i>	0613-0618
On Total Vertex Irregularity Strength of Cocktail Party Graph <i>Kristina Wijaya, Slamain, and Mirka Miller</i>	0619-0622
The Size Multipartite Ramsey Numbers $m_j(P_n, C_3)$ <i>Syafrizal Sy</i>	0623-0626
The Eccentric Digraph of an Umbrella Graph <i>Tri Atmojo Kusmayadi and Muhammad Abdul Rivai</i>	0627-0638
The Eccentric of Double Cones Graph <i>Tri Atmojo Kusmayadi and Muhammad Abdul Rivai</i>	0639-0646

MATHEMATICS EDUCATION

Computational Estimation in Grade Four: A Design Research in Indonesia <i>Al Jupri</i>	0647-0652
The Achievement of Students' Mathematical Power by Using APOS Theory <i>Elah Nurlaelah and Utari Sumarmo</i>	0653-0666

The Design of Mathematics Learning Based on Vocational Skill in Vocational High School (Use Contextual Teaching and Learning, Intergated with Vocational Problem Based Learning)	
<i>Hobri</i>	0667-0680
Some Misconceptions On Variable Ideas	
<i>Iwan Pranoto</i>	0681-0688
Factors Affecting in Indonesian Student Achievements on the International TIMSS Study	
<i>Mohammad Syaifuddin</i>	0689-0698
Critical Observation toward Propotional Reasoning Leveling	
<i>Rahmah Johar</i>	0699-0712
Designing and Implementing PMRI Learning Materials on Number for Grade 4 th Students in Palembang	
<i>Ratu Ilma Indra Putri</i>	0713-0722
A Framework for Understanding the Uses of the Internet for Teacher Professional Development	
<i>Sitti Maesuri Patahuddin</i>	0723-0734
Students' Responses to the Realistic Mathematics Teaching Approach in Junior Secondary School in Indonesia	
<i>Turmudi</i>	0735-0754
Attitude of Student Teachers of Mathematics Education Towards The Integration of ICT in Mathematics Classroom	
<i>Tutuk Narfanti and I Gusti Ngurah Darmawan</i>	0755-0764
The Nature of Discourse in PMRI Classrom: Exploring the Notion of Average	
<i>Wanty Widjaja, Hongki Julie, and Hanna Desi S.</i>	0765-0772
The “P” in PMRI: Progress and Problems	
<i>Zulkardi</i>	0773-0780

STATISTICS

Generalized Extreme Value Disrtribution Model In Option Pricing	
<i>Abdurakhman</i>	0781-0790
Procedure of Additive and Inovational Outlier Detection for Double Seasonal ARIMA Model	
<i>Alfonsus Julanto Endharta and Suhartono</i>	0791-0802
Model for Occupational Mobility	
<i>Asis Kumar Chattopadhyay</i>	0803-0822
The Hull-White One-Factor and Two-Factor Models in Approximating The Zero-Coupon Bond Prices	
<i>Bevina D. Handari, Irwanto, Ayodya R.H., and Novita M.</i>	0823-0836
Cointegration Approach on Estimation Model of Export and Import Agricultural Product	
<i>Brodjol Sutijo</i>	0837-0844
Spline Estimator in Homoscedastic Multi-Response nonparametric Regression Model	
<i>Budi Lestari, I Nyoman Budiantara, Sony Sunaryo, and Muhammad Mashuri</i>	0845-0854
Study on Least Squares Estimation and Its Properties in the GSTAR Model	
<i>Budi Nurani Ruchjana, H.P.Lopuhaa, and Svetlana A. Borovkova</i>	0855-0862
Value at Risk Calculation for Single Asset Returns Series Using Stable Distribution	
<i>Dedi Rosadi</i>	0863-0870

On Creating Truncated Weibull Distribution Module in WinBUGS and Its Use Bayesian Frontier Function Modeling <i>Dedy Dwi Prastyo, Nur Iriawan, and Yuni Wulandari</i>	0871-0884
Comparison Between ARIMA and ARAR Forecasting Method Applide for Quarterly Deaths Data In Australia <i>Dewi Anggraini</i>	0885-0896
The Reliability Monitoring Method for Wear Failure Mode <i>Erni D. Sumaryatie and Indwiarti</i>	0897-0904
The Permutation Test for Quantitative Trait Locit (QTL) Mapping <i>Farid M. Afendi and Dea Rynanda Putri</i>	0905-0912
Monte Carlo Simulation Method with Control Variate for Indonesian Option Pricing under Arithmetic Average <i>Gunardi</i>	0913-0920
Weighted Spline Estimator in Heteroscedastic Nonparametric Regression for Longitudinal Data <i>I Nyoman Budiantara, Budi Lestari, and Anna Islamiyati</i>	0921-0934
Fuzzy Regression Analysis with Symmetrical Fuzzy Dependent Variable <i>Iqbal Kharisudin and Subanar</i>	0935-0950
Cencored Data Modelling Using Bivariate Classical and Tobit Regression Analysis <i>Ismaini Zain, Dwiatmono Agus Widodo, and Is Anjarwati</i>	0951-0958
Bivariat Binary Logistic Regression Modelling on the Economic Children Labor Force Participation <i>Ismaini Zain, Hera Hendra Permana, and Siswadi</i>	0959-0968
Effect of Misused Distribution and Correlation on Mixed Logit and Probit Models for Multivariate Binary Response <i>Jaka Nugraha, Suryo Guritno, and Sri Haryatmi</i>	0969-0978
Binary Response Nonparametric Regression Model and Its Application in University Graduation <i>Jerry Dwi Trijoyo Purnomo and Suhartono</i>	0979-0986
Nonparametric Conditional Density Estimation <i>Kartiko</i>	0987-0992
Modeling Indonesian LQ45 Stock Market Index Volatility (Application of Garch and Bayesian Garch) <i>M. Arbi Hadiyat</i>	0993-1000
Bivariate Atribute Control Charts Based on Log Linear Model <i>Muhammad Mashuri, and Wibawati</i>	1001-1010
A Joint Econometric Model of Macroeconomic and Term Structure Dynamics for Indonesian Government Bond Yied Rates <i>Muhammad Syamsuddin, Lienda Noviyanti, and Rina Rahmawati</i>	1011-1020
False Discovery Rate Control in Detecting QTL in Categorical Scale <i>Rahmat Hidayatullah and Farit Mochamad Afendi</i>	1021-1034
Outlier Detection by Dffits for Robust Regression Modeling <i>Rokhana Dwi Bakti and Sutikno</i>	1035-1040
Geographically Weighted Poisson Regression Model <i>Salmon Notje Aulele and Purhadi</i>	1041-1048
Assessing Gender Bias for Good Consumption Using Semiparametric Regression <i>Sri Haryatmi Kartiko</i>	1049-1056
Comparison Between ARIMA, Transfer Function, and ASTAR Models for Forecasting Rainfall Data In Indonesia <i>Suhartono, Sutikno, B. W. Otok, and Setiawan</i>	1057-1068

Mean-VaR Portofolio Under CAPM with Lagged and non Constant Volatility <i>Sukono, Subanar, and Dedi Rosadi</i>	1069-1078
Robust Decline Curve Analysis <i>Sutawanir Darwis, Budi Nurani Ruchjana, and Asep Kurnia Permadi</i>	1079-1086
Prediction of Monthly Rainfall Characteristics based on Climate Indices Using Multi Input Transfer Functions <i>Sutikno, B.W. Otok, Suhartono, Setiawan, and A.S. Endharta</i>	1087-1096
On Choosing of Optimal Bandwidth for Fourier Series Estimator in Multiresponse Non Parametric Regression <i>Wahyu Wibowo, Agustini Tripena, I Nyoman Budiantara, and Ihsan</i>	1097-1108
A Functional Central Limit Theorem for Residual Partial Sums Process of Heteroscedastic Spatial Linear Regression Model <i>Wayan Somayasa</i>	1109-1118
Control Chart Variability of Non Parametric <i>Wibawati and Muhammad Mashuri</i>	1119-1130
Application of Spatial Scan Statistics on Tuberculosis Hotspot Detection in Indonesia <i>Yekti Widyaningsih and Siti Nurrohmah</i>	1131-1140
The Appropriate Weighted for Forecasting of MA(1) Process Based On Fuzzy Time Series <i>Zuhaimy Ismail, Muhammad Hisyam Lee, and Riswan Efendi</i>	1141-1152
Weighted Fuzzy Time Series Model for Malaysian-Indonesian Stock Index Prediction <i>Zuhaimy Ismail, Muhammad Hisyam Lee, Riswan Efendi, Suhartono, and Adlina Abdul Samad</i>	1153-1170

A MAX-PLUS ALGEBRA APPROACH TO CRITICAL PATH ANALYSIS IN THE PROJECT NETWORK WITH FUZZY ACTIVITY TIMES

M. ANDY RUDHITO, SRI WAHYUNI, ARI SUPARWANTO, AND F. SUSILO

Abstract. The activity times in a project network are seldom precisely known, and then could be represented into the fuzzy number, that is called fuzzy activity times. This paper aims to determine the fuzzy earliest starting times, fuzzy total duration time and fuzzy critical path using max-plus algebra approach. The finding shows that the project network with fuzzy activity times can be represented as a matrix over fuzzy number max-plus algebra. The project network dynamics can be represented as a system of fuzzy number max-plus linear equations. From the solutions of the system we can determine the fuzzy earliest starting times and fuzzy total duration time. The fuzzy critical path with a certain degree of critically can be determine through an interval critical path determination. We know that an alpha-cut of the fuzzy activity times is an interval activity time. The interval critical path can be determined through a crisp critical path determination. Meanwhile, the crisp critical path can be determined via a computation using max-plus algebra approach.

Key words and Phrases : max-plus algebra, project network, fuzzy activity times, fuzzy critical path.

1. Introduction

Let $\mathbf{R}_\varepsilon := \mathbf{R} \cup \{\varepsilon\}$ with \mathbf{R} thr set of all real numbers and $\varepsilon := -\infty$. In \mathbf{R}_ε defined two operations : $\forall a, b \in \mathbf{R}_\varepsilon$, $a \oplus b := \max(a, b)$ and $a \otimes b := a + b$. We can show that $(\mathbf{R}_\varepsilon, \oplus, \otimes)$ is a commutative idempotent semiring with neutral element $\varepsilon = -\infty$ and unity element $e = 0$. Moreover, $(\mathbf{R}_\varepsilon, \oplus, \otimes)$ is a semifield, that is $(\mathbf{R}_\varepsilon, \oplus, \otimes)$ is a commutative semiring, where for every $a \in \mathbf{R}$ there exist $-a$ such that $a \otimes (-a) = 0$. Thus, $(\mathbf{R}_\varepsilon, \oplus, \otimes)$ is a *max-plus algebra*, and is written as \mathbf{R}_{\max} . One can define $x^{\otimes 0} := 0$, $x^{\otimes k} := x \otimes x^{\otimes k-1}$, $\varepsilon^{\otimes 0} := 0$ and $\varepsilon^{\otimes k} := \varepsilon$, for $k = 1, 2, \dots$

The operations \oplus and \otimes in \mathbf{R}_{\max} can be extend to the matrices operations in $\mathbf{R}_{\max}^{m \times n}$, with $\mathbf{R}_{\max}^{m \times n} := \{A = (A_{ij}) \mid A_{ij} \in \mathbf{R}_{\max}, \text{ for } i = 1, 2, \dots, m \text{ and } j = 1, 2, \dots, n\}$, the set of all matrices over max-plus algebra. Specifically, for $A, B \in \mathbf{R}_{\max}^{n \times n}$ we define $(A \oplus B)_{ij} = A_{ij} \oplus B_{ij}$ and $(A \otimes B)_{ij} = \bigoplus_{k=1}^n A_{ik} \otimes B_{kj}$. We also define matrix

$$E \in \mathbf{R}_{\max}^{n \times n}, (E)_{ij} := \begin{cases} 0, & \text{if } i = j \\ \varepsilon, & \text{if } i \neq j \end{cases} \text{ and } \mathcal{E} \in \mathbf{R}_{\max}^{m \times n}, (\mathcal{E})_{ij} := \varepsilon \text{ for every } i \text{ and } j.$$

For any matrices $A \in \mathbf{R}_{\max}^{n \times n}$, one can define $A^{\otimes 0} = E_n$ and $A^{\otimes k} = A \otimes A^{\otimes k-1}$ for $k = 1, 2, \dots$. For any weighted, directed graph $G = (V, A)$ we can define a matrix $A \in \mathbf{R}_{\max}^{n \times n}$, $A_{ij} = \begin{cases} w(j, i), & \text{if } (j, i) \in \mathcal{A} \\ \varepsilon, & \text{if } (j, i) \notin \mathcal{A}. \end{cases}$, called the *weight-matrix* of graph G . Further details about max-plus algebra, matrix and graph can be found in Baccelli *et.al* (2001) and Rudhito A (2003).

A method to analyze the critical path in the project network with crisp (real) activity times, using max-plus algebra approach had been developed in Rudhito, *et.al.* (2008^b). The followings are some result in brief. A *project network* S with crisp activity times, is a directed, strongly connected, acyclic, crisp weighted graph $S = (\mathcal{V}, \mathcal{A})$, with $V = \{1, 2, \dots, n\}$ such that if $(i, j) \in \mathcal{A}$, then $i < j$. Let x_i^e is be *crisp-earliest starting time* from node i and $\mathbf{x}^e = [x_1^e, x_2^e, \dots, x_n^e]^T$. For a project network with crisp activity times, with n nodes and A the weight matrix of graph of the networks, then

$$\mathbf{x}^e = (E \oplus A \oplus \dots \oplus A^{\otimes n-1}) \otimes \mathbf{b}^e = A^* \otimes \mathbf{b}^e \quad (1)$$

with $\mathbf{b}^e = [0, \varepsilon, \dots, \varepsilon]^T$. Furthermore x_n^e is the total crisp duration time of the project. Let x_i^l is be *crisp latest completion time* for all activities that come to node i and $\mathbf{x}^l = [x_1^l, x_2^l, \dots, x_n^l]$. For the project network above, vector

$$\mathbf{x}^l = -((A^T)^* \otimes \mathbf{b}^l) \quad (2)$$

with $\mathbf{b}^l = [\varepsilon, \varepsilon, \dots, -x_n^e]^T$. From (1), (2) and weight-matrix A , we can determine the *total slack times* for activities $(i, j) \in \mathcal{A}$

$$TS_{ij} = x_j^l - x_i^e - A_{ij}. \quad (3)$$

Definition 1.1 An activity $(i, j) \in \mathcal{A}$ is called a **crisp-critical activity** if $TS_{ij} = 0$. A path $p \in P$ is called **crisp-critical path** if all activities belonging to p are critical activities, where P is the set of all path in network from node 1 to node n .

An approach to analyze the critical path in the project network with interval activity times had been developed in Chanas & Zielinski (2001) with the classical approach. Meanwhile, we will develop on the max-plus algebra approach. We will use some results in Chanas & Zielinski (2001). The followings are some result in brief. A *project network* S with interval activity times, is a directed, strongly connected, acyclic, interval weighted graph $S = (\mathcal{V}, \mathcal{A})$, with $V = \{1, 2, \dots, n\}$ such that, if $(i, j) \in \mathcal{A}$, then $i < j$.

Definition 1.2 A path $p \in P$ is called an **interval-critical path** in S if there exist a set of times $A_{ij}, \bar{A}_{ij} \in [\underline{A}_{ij}, \bar{A}_{ij}]$, $(i, j) \in \mathcal{A}$, such that p is crisp-critical, after replacing the interval times A_{ij} with crisp activity time A_{ij} , $(i, j) \in \mathcal{A}$.

Theorem 1.2 [3] A path $p \in P$ is interval-critical in S if and only if crisp-critical in S , in which the interval activity times $A_{ij} \in [\underline{A}_{ij}, \bar{A}_{ij}]$, $(i, j) \in \mathcal{A}$, have been replace

with crisp activity times A_{ij} determined by means of the following formula

$$A_{ij} = \begin{cases} \bar{A}_{ij}, & \text{if } (i, j) \in p, \\ \underline{A}_{ij}, & \text{if } (i, j) \notin p. \end{cases} \quad (4)$$

We will discuss the determining the fuzzy earliest starting times, fuzzy total duration time and fuzzy critical path using max-plus algebra approach. Basically, the notion of the determining fuzzy earliest starting times, fuzzy total duration time is analog with the method in Rudhito, *et.al.* (2008^b), where the activity times is crisp. We will develop for the fuzzy activity times. We replace the deterministic activity times with the fuzzy activity times. The discussion on the fuzzy critical path using will be based on the result in Rudhito, *et.al.* (2008^b) and Chanas & Zielinski (2001, 2002) like above. We also give some examples for illustration.

We will used some concepts and result on the fuzzy set and fuzzy number, that can be found in Zimmermann, H.J., (1991), Lee, K.H. (2005) and Susilo, F. (2006), fuzzy number max-plus algebra and matrix over fuzzy max-plus algebra in Rudhito, *et.al.* (2008^a) and iterative system of fuzzy number max-plus linear equations in Rudhito, *et.al.* (2008^c)

2. Main Results

Definition 2.1 A project networks with fuzzy activity times \tilde{S} is a directed, connected, acyclic and fuzzy number-valued weighted graph $\tilde{S} = (\mathcal{V}, \tilde{A})$, with $\mathcal{V} = \{1, 2, \dots, n\}$ such that, if $(i, j) \in \tilde{A}$, then $i < j$.

In this project, an arch represent an *activity*, fuzzy number-valued weight of an arch represent an *activity times*, so an fuzzy number-valued weight in this networks is a nonnegative fuzzy number (where its α -cut is an nonnegative interval).

Let \tilde{x}_i^e be the fuzzy earliest starting time from node i .

$$\tilde{A}_{ij} = \begin{cases} \text{fuzzy activity times from } j \text{ to } i, & \text{if } (j, i) \in \tilde{A} \\ \tilde{\varepsilon}, & \text{if } (j, i) \notin \tilde{A}. \end{cases}$$

We assume that $\tilde{x}_1^e = \tilde{0}$ and with max-plus fuzzy number notation we have

$$\tilde{x}_i^e = \begin{cases} \tilde{0}, & \text{if } i = 1 \\ \bigoplus_{1 \leq j \leq n} (\tilde{A}_{ij} \otimes \tilde{x}_j^e), & \text{if } i > 1. \end{cases} \quad (5)$$

Let \tilde{A} be the fuzzy number weight matrix of the fuzzy number-valued weighted graph of the networks, $\tilde{\mathbf{x}}^e = [\tilde{x}_1^e, \tilde{x}_2^e, \dots, \tilde{x}_n^e]^T$ and $\tilde{\mathbf{b}}^e = [0, \tilde{\varepsilon}, \dots, \tilde{\varepsilon}]^T$, equation (5) can be written in an iterative system of fuzzy max-plus linear equations

$$\tilde{\mathbf{x}}^e = \tilde{A} \otimes \tilde{\mathbf{x}}^e \oplus \tilde{\mathbf{b}}^e. \quad (6)$$

Since the project networks is acyclic directed graph, then there are no circuit, so according to the result in Rudhito, *et.al.* (2008^c), \tilde{A} is definite. And then also according to the result in Rudhito, *et.al.* (2008^c), the fuzzy number vector $\tilde{\mathbf{x}}^e$ with $\tilde{x}_i^e = \bigcup_{\alpha \in [0,1]} \tilde{c}_i^\alpha$ where \tilde{c}_i^α is a fuzzy set in \mathbf{R} with membership function $\mu_{\tilde{c}_i^\alpha}(x) = \alpha \chi_{(A^* \otimes \mathbf{b}^e)_i^\alpha}(x)$, where $\chi_{(A^* \otimes \mathbf{b}^e)_i^\alpha}$ is a characteristic function of set $(A^* \otimes \mathbf{b}^e)_i^\alpha$, is a unique solution of the system (6), that is the vector of the fuzzy earliest starting time for each node in the project.

Notice that $(\tilde{A}^*)_n$ is a maximum weigh of path form the start node to the end node, hence \tilde{x}_n^e is the fuzzy total duration time of the project. We summarize the description above in the Theorem 2.1.

Theorem 2.1 *Given a project network with fuzzy activity times, with n node and \tilde{A} is the weight matrix of the fuzzy number-valued weighted graph of networks. The vector $\tilde{\mathbf{x}}^e$ is given by fuzzy number vector with $\tilde{x}_i^e = \bigcup_{\alpha \in [0,1]} \tilde{c}_i^\alpha$ where \tilde{c}_i^α is a fuzzy set in \mathbf{R} with membership function $\mu_{\tilde{c}_i^\alpha}(x) = \alpha \chi_{(A^* \otimes \mathbf{b}^e)_i^\alpha}(x)$ and $\chi_{(A^* \otimes \mathbf{b}^e)_i^\alpha}$ is a characteristic function of set $(A^* \otimes \mathbf{b}^e)_i^\alpha$, with $\tilde{\mathbf{b}}^e = [0, \tilde{\varepsilon}, \dots, \tilde{\varepsilon}]^T$.*

Proof: see description above. ■

Before we give an example, we recall about a special types of fuzzy number. A *triangular fuzzy number* \tilde{a} , which is written as $\text{TFN}(a_1, a, a_2)$ or (a_1, a, a_2) , is a fuzzy number with membership function

$$\mu_{\tilde{a}}(x) = \begin{cases} \frac{x-a_1}{a-a_1}, & a_1 \leq x \leq a \\ \frac{a_2-x}{a_2-a}, & a \leq x \leq a_2 \\ 0, & \text{others} \end{cases}.$$

The *support* of \tilde{a} is an open interval (a_1, a_2) and its α -cut is

$$a^\alpha = \begin{cases} [(a-a_1)\alpha + a_1, -(a_2-a)\alpha + a_2] & , \alpha \in (0,1] \\ [a_1, a_2] & , \alpha = 0 \end{cases}.$$

Example 2.1 Consider the project network in Figure 2.1.

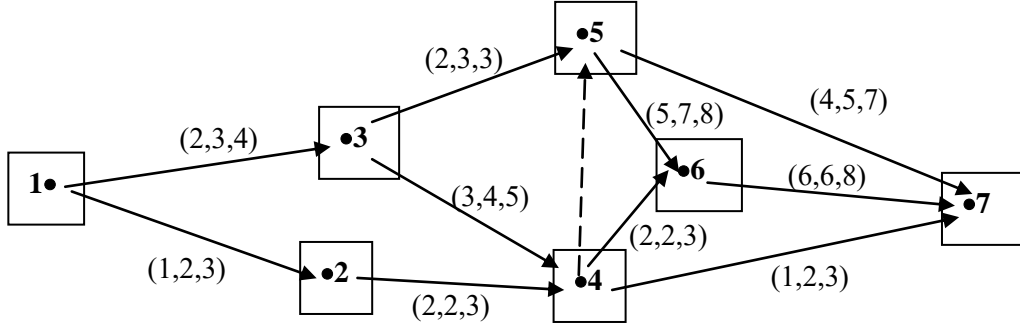


Figure 2.1 Project Network

We have

$$\tilde{A} = \begin{bmatrix} \tilde{\varepsilon} & \tilde{\varepsilon} & \tilde{\varepsilon} & \tilde{\varepsilon} & \tilde{\varepsilon} & \tilde{\varepsilon} & \tilde{\varepsilon} \\ (1,2,3) & \tilde{\varepsilon} & \tilde{\varepsilon} & \tilde{\varepsilon} & \tilde{\varepsilon} & \tilde{\varepsilon} & \tilde{\varepsilon} \\ (2,3,4) & \tilde{\varepsilon} & \tilde{\varepsilon} & \tilde{\varepsilon} & \tilde{\varepsilon} & \tilde{\varepsilon} & \tilde{\varepsilon} \\ \tilde{\varepsilon} & (2,2,3) & (3,4,5) & \tilde{\varepsilon} & \tilde{\varepsilon} & \tilde{\varepsilon} & \tilde{\varepsilon} \\ \tilde{\varepsilon} & \tilde{\varepsilon} & (2,3,3) & \tilde{0} & \tilde{\varepsilon} & \tilde{\varepsilon} & \tilde{\varepsilon} \\ \tilde{\varepsilon} & \tilde{\varepsilon} & \tilde{\varepsilon} & (2,2,3) & (5,7,8) & \tilde{\varepsilon} & \tilde{\varepsilon} \\ \tilde{\varepsilon} & \tilde{\varepsilon} & \tilde{\varepsilon} & (1,2,3) & (4,5,7) & (6,6,8) & \tilde{\varepsilon} \end{bmatrix}.$$

Using MATLAB computer program, we have bounds of α -cut of components of vector \tilde{x}^e and the sketch of them, for $\alpha = 0, 0.05, \dots, 0.95, 1$, is the Figure 2.2 bellow.

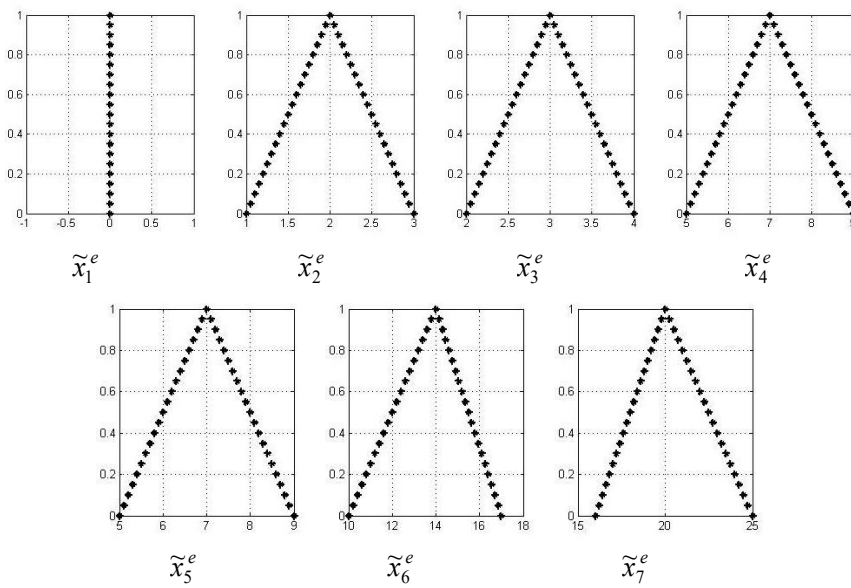


Figure 2.2 Graph of the bounds of α -cut of components of \tilde{x}^e .

From the Figure 1, we have $\tilde{x}_1^e = \text{TFN}(0, 0, 0)$, $\tilde{x}_2^e = \text{TFN}(1, 2, 3)$, $\tilde{x}_3^e = \text{TFN}(2, 3, 4)$, $\tilde{x}_4^e = \text{TFN}(5, 7, 9)$, $\tilde{x}_5^e = \text{TFN}(5, 7, 9)$, $\tilde{x}_6^e = \text{TFN}(10, 14, 17)$ and $\tilde{x}_7^e = \text{TFN}(16, 20, 25)$. The fuzzy total duration time of the project is $\tilde{x}_7^e = \text{TFN}(16, 20, 25)$.

In the fuzzy critical path discussion below, we will talk about path degree of criticality. We will review some definition and theorem, which are develop from Chanas & Zielinski (2001)

Definition 2.2 The fuzzy set \tilde{P} in set P with the membership function $\mu_{\tilde{P}}: P \rightarrow [0,1]$ determined by formula

$$\mu_{\tilde{P}}(p) = \sup_{\substack{A_{ij} \in \mathbf{R}^+, (i,j) \in \tilde{A} \\ \text{and } p \text{ is crisp-critical with} \\ \text{crisp activity times} \\ \text{equal to } A_{ij}, (i,j) \in \tilde{A}}} \min_{(i,j) \in A} \mu_{\tilde{A}_{ij}}(A_{ij}), p \in P$$

is called **fuzzy-critical path** in \tilde{S} with degree $\mu_{\tilde{P}}(p)$.

Definition 2.3 The scalar $\alpha \in (0, 1]$ is called **feasible under the path** p if p is interval-critical in the network \tilde{S} with interval activity times $A_{ij} = A_{ij}^\alpha$, where A_{ij}^α are α -cut of fuzzy activity times \tilde{A}_{ij} .

Theorem 2.2 The following equality holds :

$$\mu_{\tilde{P}}(p) = \sup \{ \alpha \mid \alpha \text{ is fisible value under the path } p \in P \}.$$

Proof: Obvious, according to the Decomposition Theorem in fuzzy set. ■

Definition 2.4 A path p is said to have **degree of criticality** $\mu_{\tilde{P}}(p) = 0$ if $\alpha = 0$ is not feasible under the path p .

Now we present an algorithm for computing the criticality degree $\mu_{\tilde{P}}(p)$ of path $p \in P$. This algorithm is developed from an algorithm in Chanas & Zielinski (2001). The algorithm is based on the idea bisection of the interval $[0, 1]$ of possible values of α to compute the maximal feaseble α_{\max} . For testing of feasibility, we used (4) and determining crisp critical path, we used (1), (2) and (3).

Algorithm 2.1 (Determining the path degree of criticality)

Step 1 :

Assign $k := 0$.

Step 2 :

Tes feasibility $\alpha = 0$ under the path p . If it is not, then $\alpha_{\max} = 0$ and go to Step 6.

Step 3 :

Tes feasibility $\alpha_k = 1$ under the path p . If it is feasible under the path p , then $\alpha_{\max} = 1$ and go to Step 6.

Step 4 :

Assign $k := k + 1$.

$$\alpha_k := \begin{cases} \alpha_{k-1} + \frac{1}{2^k}, & \text{if } \alpha_{k-1} \text{ feasible} \\ \alpha_{k-1} - \frac{1}{2^k}, & \text{if } \alpha_{k-1} \text{ not feasible.} \end{cases}$$

Tes feasibility α_k under the path p . If it is feasible assign $\alpha_{\max} = \alpha_k$.

Step 5:

If $k < K$, then go to step 4.

Step 6:

Assign $\mu_{\bar{p}}(p) = \alpha_{\max}$. Stop.

With: $K \geq N / {}^{10}\log 2$, with absolute error of computation 10^{-N} .

Example 2.2 Consider the project network in Example 2.1. Applying Algorithm 2.1 we have obtained results which are listed in Table 2.1. The path degree of criticality have been computed with accuracy 10^{-2} .

Table 2.1 The path degree of criticality

No	Path p	$\mu_{\bar{p}}(p)$
1	1→3→5→7	0
2	1→3→5→6→7	0,0039
3	1→3→4→5→7	0
4	1→3→4→5→6→7	1
5	1→3→4→6→7	0
6	1→3→4→7	0
7	1→2→4→5→7	0
8	1→2→4→5→6→7	0,25
9	1→2→4→6→7	0
10	1→2→4→7	0

References

[1] F. Bacelli, *et al.*, *Synchronization and Linearity*, John Wiley & Sons, New York, 2001.

[2] S. Chanas, S., P. Zielinski, P, Critical path analysis in the network with fuzzy activity times. *Fuzzy Sets and Systems*. **122**. 195–204., 2001.

[3] S. Chanas, S., P. Zielinski, P, The computational complexity of the critical problems in a network with interval activity times. *European Journal of Operational Research*, **136**. 541–550., 2002.

[4] K.H. Lee, *First Course on Fuzzy Theory and Applications*, Springer-Verlag, Berlin Heidelberg, 2005.

[5] M. A. Rudhito, Sistem Linear Max-Plus Waktu-Invariant, Tesis: Program Pascasarjana Universitas Gadjah Mada, Yogyakarta, 2003.

[6] M. A. Rudhito, S. Wahyuni, A. Suparwanto, and F. Susilo, Sistem Persamaan Linear Max-Plus Bilangan Kabur. *Prosiding Konferensi Nasional Matematika XIV FMIPA UNSRI Palembang* 24 – 27 Juli, 2008^a.

[7] M. A. Rudhito, S. Wahyuni, A. Suparwanto, and F. Susilo, Analisis Lintasan Kritis Jaringan Proyek dengan Pendekatan Aljabar Max-Plus. *Prosiding Seminar Nasional Matematika*, UNPAR, Bandung, Indonesia, Vol. **3**, 205 – 212., 2008^b.

- [8] M. A. Rudhito, S. Wahyuni, A. Suparwanto, and F. Susilo, Iterative System of Fuzzy Number Max-Plus Linear Equations. *Proceeding on the International Conference on Mathematics and Natural Sciences 2008*, ITB, Bandung, Indonesia, October 28 - 30, 2008^c.
- [9] F. Susilo, *Himpunan dan Logika Kabur serta Aplikasinya edisi kedua*, Graha Ilmu, Yogyakarta, 2006.
- [10] H.J. Zimmermann, *Fuzzy Set Theory and Its Applications*, Kluwer Academic Publishers, Boston, 1991.

M. ANDY RUDHITO: Ph.D Student at Department of Mathematics, Gadjah Mada University, Yogyakarta, Indonesia, Staff at Department of Mathematics Education, Sanata Dharma University, Yogyakarta, Indonesia. E-mail: arudhito@yahoo.co.id

SRI WAHYUNI: Department of Mathematics, Gadjah Mada University, Yogyakarta, Indonesia. E-mail: swahyuni@ugm.ac.id

ARI SUPARWANTO W: Department of Mathematics, Gadjah Mada University, Yogyakarta, Indonesia. E-mail: ari_suparwanto@yahoo.com

F. SUSILO: Department of Mathematics, Sanata Dharma University, Yogyakarta, Indonesia. E-mail: fsusilo@staff.usd.ac.id